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First steps *in* Modal Logic

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Introduction

This book is an introduction to modal logic, more precisely, to classically based propositional modal logic. There are few books on this subject and even fewer books worth looking at. None of these give an acceptable mathematically correct account of the subject. This book is a first attempt to fill that gap.

Apart from its mathematical clarity, some other features of the book are:

- The central concept of the book is that of a labelled transition structure, and polymodal languages are used from the beginning.
- Modal languages are viewed as a tool for analysing the properties of transition structures, not the other way round.
- There is not an overemphasis on syntactic (proof theoretic) matters.
- Nevertheless, a detailed explanation is given of the differences between the weak completeness and Kripke completeness of formal systems.
- Correspondence properties (the expressibility properties of modal languages) are stressed as an important tool.
- Bisimulations are used as a method of comparing transition structures.
- Each chapter has a decent selection of exercises and over one sixth of the book consists of a comprehensive set of solutions to these exercises.

The book is aimed primarily at a computer science readership. However there is no computer science in the book and very little material which is directly attributable to a computer science motivation. Thus the reader of the book may be interested in modal logic in its own right or because of one or several of its applications in computer science. To read the book it is not necessary to understand any of these fields of application in any great depth.

The applications and uses of modal logic are many and varied. Aspects of the subject can be found in

- The analysis of tenses
- Concurrency
- Belief logic and default logic
- Program correctness
- Power domain constructions
- Situation theory

and several other areas. To emphasize some of these in prefence to others would only restrict the possible readership. The aim of the book is simply to give a correct and concise account of the core of the subject with just a hint of the more advanced topics. The aim is not to describe the possible applications of the subject.

In order to keep the book to a reasonable size there are some important topics which the book does not attempt to cover. The book covers only propositional modal logic on a classical base. It does not consider any predicate logic versions of modal logic, nor does it consider any base logic other than classical 2-valued logic. The book does not give a detailed discussion of the various proof theoretic ramifications of modal logic. Only the simplest and most routine proof system is discussed. Nor is it concerned with the execution or implementation problems of model systems. Almost nothing is said about decision procedures for modal logic.

No doubt there are also other topics which you would have liked included, however these are for a more advanced or specialized book.

For convenience the book is divided into six reasonably sized parts. Each of these is devoted to one aspect of the subject as follows.

PART I, which covers Chapters 1 and 2, gives a survey of the required material concerning propositional logic and then introduces the family of (propositional) modal languages.

PART II, which covers Chapters 3, 4, 5, and 6, is concerned with various semantic matters. Chapter 3 introduces the structures which support the Kripke style semantics for modal languages, and Chapter 4 discusses these various semantics. The two Chapters 5 and 6 give various results which illustrate how properties of the supporting structures can be captured by modal formulas.

PART III, which covers Chapters 7, 8, 9, and 10, is concerned with proof theoretic matters. Firstly, in Chapter 7 various motivating semantic consequence relations are discussed. Then in Chapter 8, the standard proof theoretic machinery (as used in this book) is introduced and developed as far as is needed. Finally, in Chapters 9 and 10, two completeness results are proved. These show how the semantics and the proof theory interact in a nice way.

These three parts, I, II, and III, form (what should be) the basis of a first course in modal logic.

PART IV, which covers Chapters 11, 12, and 13, deals with more advanced, but still fairly basic, material. In particular, the notion of a bisimulation, the construction of models using filtrations, and the finite model property of certain formal systems are discussed.

PART V, which covers Chapters 14, 15, 16, and 17, gives four examples of more advanced, but interesting, topics. The first two examples (in Chapters 14 and 15) illustrate some of the power of modal logic, and the second two examples (in Chapters 16 and 17) illustrate some of the problems which come with this power.

PART VI consists of two Appendices. The first of these, Appendix A, contains both the first and last things you should read in the book. It is a general discussion of modal logic, from the various other introductory texts, what you can expect from this text, what you should read next, and a survey of some of the uses of modal logic. As soon as you finish this Introduction, i.e. at the end of the next sentence, you should go immediately to this Appendix. Finally, Appendix B is a fairly comprehensive set of solutions to the exercises given at the end of each chapter.

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