Hybrid Logic Tango
University of Buenos Aires
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Masterton

Arthur Prior

Born 4 December, 1914, Masterton, New Zealand.

1949 Work veers towards philosophy and logic.

1956 John Locke Lectures, Oxford.

1957 Publication of “Time and Modality”.

1967 Publication of “Past, Present and Future”.

1968 Publication of “Papers on Time and Tense”.

Died 6 October, 1969, Trondheim, Norway.

Tense Logic

Today Prior is best known for his invention of tense logic. The basic ideas of the area were created by him, and many of its most challenging problems (for example, in the logic of branching time models) trace back to his work.

Hybrid Logic

Prior also invented hybrid logic — but is work in this area is little known. An honourable exception to this neglect are the writings of Per Hasle and Peter Ohrstrom.

Why is this? Actually, it is rather puzzling. Prior wrote a lot on the subject, and it is crucial to his philosophical position.

Hybrid Logic in "Past, Present and Future"

"Past, Present and Future", Arthur Prior, Clarendon Press, 1967.

Chapter 5, Section 6, Development of the U-calculus within the theory of world states, pages 88 – 92.

Appendix B, Section 3, On the range of world-variables and the interpretation of U-calculi in world-calculi. pages 186 – 197.

Nominals are called world variables, and their interpretations world-propositions. World-calculi means "hybrid logic" (of the strong type described today) and "U-calculi" means "correspondence language".

Prior is basically interested in showing that hybrid logic can capture the entire correspondence language.

Hybrid Logic in "Papers on Time and Tense"

"Papers on Time and Tense", Arthur Prior, Clarendon Press, 1968. Over a third of the book makes use of hybrid logic. Four of the five technical papers deal with hybrid logic (the fifth is on metric tense logic)'. Two of these papers are particularly important for understanding why hybrid logic was important to Prior, and where it lead him too.

"The Logic of Ending Time", pages 98 – 115.

"Tense Logic and Logic of Earlier and Later", pages 116 – 134.

'"Quasi-Propositions and Quasi-Individuals", pages 135 – 144.

"Tense Logic for Non-Permament Existents", pages 145 – 159.

Hybrid Logic in "Worlds, Times and Selves"

"Worlds, Times and Selves", by Arthur Prior and Kit Fine, University of Massachusetts Press, 1977.

Posthumously published, essentially a collection of papers and fragments gathered together with an appendix by Kit Fine, "Worlds, Times and Selves" is one of the great might-have-beens of hybrid logic.

This book was to have dealt with "*the interplay between modal and tense logic on the one hand, and quantification theory on the other. One its man concerns was to show that modal and tense logics could stand on their own, that talk of possible worlds or instants was to be reduced these logics rather that conversely.*" From Kit Fine's Preface to WTS.

Hybrid logic crucial to Prior's thought

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- Arthur Prior invented hybrid logic to solve a philosophical difficulty.
- He then discovered that it had given him an (deeper) philosophical difficulty. Prior's nightmare.
- Nothing in his published writing fully solves the new difficulty. Perhaps he would have found answers had he lived to complete "Worlds, Times and Selves".

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Prior doesn't really *use* the ideas of hybrid logic for anything apart form solving his philosophical difficulty. Prior doesn't argue that they are of any independent logical or linguistic interest (a partial exception can be made for his paper 'Now' — which was not in the original edition of *Papers on Time and Tense*). In a nutshell, Prior doesn't really show that hybrid logic is interesting in its own right — as he so brilliantly did for tense logic.

Prior on Reichenbach

Reichenbach offered an influential analysis of tense in natural language based on the idea of reference points.

In "Past, Present, and Future", Prior is rather dismissive of Reichenbach's approach. He offers no deep criticism. His main point is that sentences like "I shall have been going to see John" require more than one reference point.

Ironically, the hybrid machinery introduced in "Past, Present and Future" allow Prior and Reichenbach's ideas to be blended seemlessly. Indeed, the hybrid machinery allows what linguists consider the deepest flaw in Reichenbachs work to be repaired ...

Prior meets Reichenbach

Structure	**Name**	**English example**	**Representatio**
E–R–S	Pluperfect	I had seen	$P(i \wedge P\phi)$
E,R–S	Past	I saw	$P(i \wedge \phi)$
R–E–S	Future-in-the-past	I would see	$P(i \wedge F\phi)$
R–S,E	Future-in-the-past	I would see	$P(i \wedge F\phi)$
R–S–E	Future-in-the-past	I would see	$P(i \wedge F\phi)$
E–S,R	Perfect	I have seen	$P\phi$
S,R,E	Present	I see	ϕ
S,R–E	Prospective	I am going to see	$F\phi$
S–E–R	Future perfect	I will have seen	$F(i \wedge P\phi)$
S,E–R	Future perfect	I will have seen	$F(i \wedge P\phi)$
E–S–R	Future perfect	I will have seen	$F(i \wedge P\phi)$
S–R,E	Future	I will see	$F(i \wedge \phi)$
S–R–E	Future-in-the-future	(Latin: abiturus ero)	$F(i \wedge F\phi)$

Shift and refer

- In essence, we have split tenses up into a 'shift' component (Prior's analysis) and a 'refer' component (Reichenbach's analysis) in a simple language.
- That is, the basically analysis is:

$$\text{SHIFTER}(\text{RESTRICTOR} \wedge \text{MATRIX})$$

- It seems that iterations of this basic pattern are typical of natural language tenses — and hybrid logic is perfect for capturing such patterns.
- For example,"I shall have been going to see John" can be represented as $F(i \wedge P(j \wedge F(\text{I-see-john})))$.

So where are we going in today's talk?

- Develop Prior's strong hybrid languages — from a modern perspective, and using modern notation.
- Discuss why Prior was interested in hybrid logic (and how his perspective differs from the modern perspective) and the philosophical difficulty hybrid logic lead him to.
- Present a modern perspective on some of the issues this raises and round off the discusssion.

Two new ingredients

Prior took his creation, tense logic and added:

- Nominals;
- The global modality (or universal modality);
- Binding nominal (state variables) with $\forall$ and $\exists$.

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- Prior asked what they were. He tried various answers, such as "the (infinite) conjunction of all the information at a world". He seems to have felt that they "embodied a description".
- I prefer a Kripke/Kaplan account — that's the natural way to think about tableau rules, the introduction of new nominals in analyses of text, and downarrow. Labelling ain't cheating!

Aside: the Q operator

Sometimes Prior works with an alternative to nominals — the Q operator.

$Q\phi$ is true at any point in any model iff there is a unique point in the model where ϕ is true.

So if we write Qp we have in effect turned the propositional variable p into a nominal.

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Box form:

$A\varphi$ means φ is true at all points in the model

Useful from an applied perspective

This modality can be useful in all sorts of ways. For example, a description logician would say that it "internalizes the TBox". We can now say things like

$$A(\text{pulis} \rightarrow \text{dogs}).$$

A computer scientist might observe that we can now specify things like:

$$A(\text{demonActivated} \rightarrow \text{printerReady}).$$

But that's not why Prior was interested in it ...

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In short, @ can be thought of as a guarded form of the global modality.

Aside : Global modality or primitive @?

- Basic hybrid logic is PSPACE complete.
- Adding E to orthodox modal logic gives us an EXPTIME complete system (Hemaspaandra), even if no nominals are added.
- That said, E can be nice to have around (for example, in description logic type applications) and its a standard tool in modern hybrid logic.
- Incidentally, though it smashes locality wide open, E is a beautifully behaved modality, largely because it is, in curious way, internal. See Chapter 7 of "Modal logic", by Blackburn, de Rijke and Venema, Cambridge University Press, 2001.

Binding with $\forall$ and $\exists$

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(Trichotomy: given any two points in the model, either they are related or they are equal.)

Syntax

- Choose a denumerably infinite set $\mathrm{SVAR} = \{x, y, z \ldots\}$, the set of state variables, disjoint from PROP, NOM and MOD.
- The strong hybrid language (over PROP, NOM, MOD and SVAR) is defined as follows:

$$\begin{aligned} \mathrm{WFF} \quad := \quad & x \mid i \mid \mathrm{p} \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \\ & \mid \varphi \rightarrow \psi \mid \langle \mathrm{M} \rangle \varphi \mid [\mathrm{M}] \varphi \mid E\varphi \mid \exists x \varphi \end{aligned}$$

- We define $\forall x \varphi$ to be $\neg \exists x \neg \varphi$. We define $@_i \varphi$ to be $E(i \wedge \varphi)$.

Satisfaction definition

$$\mathcal{M}, g, w \Vdash x \quad \text{iff} \quad w = g(x) \text{ where } x \in \text{SVAR}$$

$$\mathcal{M}, g, w \Vdash E\varphi \quad \text{iff} \quad \exists w'(\mathcal{M}, g, w' \Vdash \varphi)$$

$$\mathcal{M}, g, w \Vdash \exists x.\varphi \quad \text{iff} \quad \mathcal{M}, g', w \Vdash \varphi, \text{ where } g' \overset{x}{\sim} g.$$

The fifth clause gives the obvious "classical" definition to $\exists$: we simply rebind x to some state in a model. (As before, he notation $g' \overset{x}{\sim} g$ means that g' is the assignment that differs from g, if at all, only in what it assigns to x.)

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For a start, we have $\downarrow$:

$$\downarrow x.\varphi \quad =_{def} \quad \exists x(x \wedge \varphi).$$

So the new binders are at least as strong as $\downarrow$. Note that $\downarrow$ is essentially a guarded form of $\exists$.

The examples on the previous slide show that the new language is strictly stronger, as sentences in the downarrow language are invariant under generated submodels.

The Hybrid translation

Indeed, we now have the expressive power needed to capture the entire first-order temporal language:

$$
\begin{array}{lcl}
HT(xRy) & = & @_x \Diamond y \\
HT(\mathsf{P}x) & = & @_x p \\
HT(x = y) & = & @_x y \\
HT(\neg\varphi) & = & \neg HT(\varphi) \\
HT(\varphi \wedge \psi) & = & HT(\varphi) \wedge HT(\psi) \\
HT(\exists v \varphi) & = & \exists v HT(\varphi) \\
HT(\forall v \varphi) & = & \forall v HT(\varphi).
\end{array}
$$

(This translation was known to Prior in the mid 1960s.)

Note: @ is needed for first-order expressivity

- Without @, we do not have full first-order logic.
- In fact, $\exists$ is "nearly local". When @ is dropped form the language, formulas are preserved under "generated submodels + extra point". ("Hybrid Logic", Blackburn and Seligman, JoLLI, 1995).
- That is, in the strong hybrid logic, classical quantification is "factored" a "binding" step, and "carry out evaluation step".
- This fragment has turned up in the setting of feature logic (where it is NP-complete).

Moreover $\downarrow + E$ yield first-order expressivity

We have:

$$\exists x\varphi \quad =_{def} \quad \downarrow y.E \downarrow x.E(y \wedge \varphi)$$

(Here y does not occur in φ.)

The hybrid hierarchy

- Orthodox modal logic.
- Basic hybrid logic.
- Add E **OR** add $\downarrow$.
- Add E **AND** add $\downarrow$ (= Prior's strong language).

How interesting is Prior's strong language?

- Because it is essentially a notational variant of the correspondence language, there's nothing much to learn about it's meta-theory: we can "read off" the results we need from first-order logic.
- However it differs in subtle ways, and this is sometimes useful in analyzing weaker hybrid languages: the prenex hierarchy ascends more slowly. This can be interesting.
- Nonetheless, the history of hybrid logic since this time is how to make the idea of nominals, shifting points of evaluation and names to states work in weaker logics. This process eventually lead to the basic hybrid language and $\downarrow$.

But *why* did Prior explore hybrid logic?

- Strongly influenced by natural language. Believed that the modal perspective with its internal view captured genuine temporal logic.
- But his was not a modern natural language semantics perspective — he was in concrete semantic problems principally as a way into phislosophical issues.
- For example, he did not see that nominals allowed a beautiful reconciliation between his views and Reichenbach's views.
- Rather, his interest in hybrid logic was driven by his fundamental philosophical conviction concerning time and tense....

A series and B series

A series: The flow of time from past, through present, to future. Internal. Situated. Arguably the way we experience time. Tense logic was intended to reflect its structure.

B series: Time as a set of instants ordered by $<$. External. Eternal. Arguably not the way we experience time. First-order logic (the correspondence language) is the tool for describing it.

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Modern hybrid logic (indeed modern modal logic) shares Prior's appreciation of the internal perspective. But it comes with no built-in ontological predjudices: models (graphs) are viewed as a playground for ontological experimentation.

Logic not model theoretic for Prior

It's worth stressing that Prior's view of logic is *very* different from the one now regarded as standard. It's not model theoretic.

- Logic for Prior was the ground floor. A "model theoretic semantics" for a language was at best a useful heuristic. Tense logic was conceived semantically, but required no — indeed could have no — genuine conceptual underpinnings.
- This view of logic reasonably common: Frege, (Quine?), Martin Löf type theorists.
- But it is diametrically opposed to stance taken in contemporary hybrid (and modal, and description) logic. Remember the slogans we started with!

Problem: Tense Logic too weak

- Unfortunately, as Prior was well aware, tense logic (the A-series language) is weaker that the B-series language (the first-order correspondence language). (As we now know, it only gives us the bisimulation invariant fragment of the first-order correspondence language.)
- This was unacceptable — Prior believed that A-series talk should be able to ground B-series talk. That is, he wanted an A-series language strong enough to swallow all of B-series talk.
- What to do?

Strong hybrid logic

- Prior hybridized. He added nominals, the universal modality, and allowed quantification across nominals, creating the strong hybrid language we have just discussed.
- And he gave the hybrid translation — thereby showing that B-series talk was reducible to A-series talk.
- Was globality an issue? Yes, but Prior thought he had solved this satisfactorily. For example, over realistic models of time, A is definable in terms of the tense operators (he called this fourth grade tense logic).

Moreover no more instants...

And hybridization offered more:

> *...a world-state proposition in the tense-logical sense is simply index of an instant; indeed, I would like to say that it is an instant in the only sense in which 'instants' are not highly fictitious entities (PTT page 188).*

- That is, temporal ontology has been banished: only propositions remain.
- In many respects, his is an attractive and curiously modern view, with echos of situation theory. It is instructive to read Prior, and consciously substitute the word "information" for "proposition".

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- For example it could be used to reason about people and their properties and relations. (Prior actually discusses that — arguably he was the first person to do description logic.)
- So not only temporal talk could be reduced to hybrid logic, any kind of first order discourse could be.
- But Prior had wanted to draw a fundamental distinction between the two modes of discourse. Suddenly nothing remains of this — Prior's nightmare.

What to do?

In papers of time and tense he explores two options. Here's the first:

> *Philosophically where do we go from here? We could turn the tables on the objectors to tense logic by saying that only are 'instants' not genuine individuals there no genuine individuals, only certain propositions that can be formally treated as if they they were individuals. [PTT, page 141]*

Interesting, and rhetorically attractive ("embrace the nightmare") — but as Prior remarks, most people would probably find this unpalatable. And anyway it doesn't solve his problem as it fails to draw a distinction.

Or this

> *So far as I can see, there is nothing philosophically disreputable in saying that (i) persona just are genuine individuals, so that their figuring as individuals in a first-order theory needs no explaining (this first-order theory being, on the contrary, the only way of giving sense to its 'modal' counterpart), whereas (ii) instants are not genuine individuals, so that their figuring as values of individual variables does need explaining, and it the related 'modal logic' (tense logic) which gives the first-order logic what sense it has. [PTT, page 142]*

In *Papers on Time and Tense* he does not get much farther than outlining the options.

Worlds, Times and Selves (I)

Logicians have tended to welcome the presentation of modal logic as an artificially truncated bit of predicate calculus because we know all about predicate calculus, or at all events know an enormous lot about it, whereas modality is a comparatively obscure and unfamiliar field. And even philosophically, it might be said, it is in general pretty clear what is going on in predicate calculus, but not very clear what is going on in modal logic or even tense logic. [WTS page 56.]

Worlds, Times and Selves (II)

It is not as simple as this. What we can do with first-order predicate logic in toto is indeed plain enough; but its uniform monadic fragment? Formally, this fragment is no doubt of some interest; for example, unlike the full first-order predicate calculus it is decidable. But what is its philosophical interest? The question, I think partly boils down to this one: What would a philosphically privileged individual be? And to this question, modal and tense logic possibly provide an answer. It is not that modal logic or tense logic is an artifically truncated uniform monadic first-order predicate calculus; the later, rather, is artificiallly expanded modal or tense logic.

Worlds, Times and Selves (III)

Other interesting answer sketches in WTS — but nothing conclusive.
A great deal of effort devoted to constructing various formal systems, and comparing them — but rather little philsophical discussion in the fragments we have of it.

What can we say from a modern perspective?

- Key difference is the primacy of model theoretic perspective.
- The Amsterdam perspective on modal logic (including hybrid logic) is that we are engaged in an enterprise of exploring fragments of (usually classical) logic from a model theoretic perspective.
- Modal logics are not isolated formal systems. Indeed the goal is to find alternative ways of talking about relational structures, and natural sublogics.
- But why is this model theoretic perspective interesting?

Logic returning to its roots

- Antiquity to late 19th century: logic firmly linked to language, knowledge, and cognition. A tool for exploring such issues.
- 20th century. Logic becomes mathematical, and is applied in various branches of mathematics.
- Recent developments: Logics for knowledge representation, logics for natural language semantics, logics for computation ...
- Logic returning to it roots — but stronger than ever. Turned into a genuinely useful tool because of the 20th century mathematical turn.
- And arguably the model-theoretic perspective is the key.

Logic the Janus-faced science

- Key insight: to think about language, and representational issues, we need to make two abstractions: ontology (models do this for us) and language (we have a choice of how to talk about structures.
- Models an appropriate level for thinking about "softer" problems, such as those from natural language semantics and knowledge representations. (We often don't know much more than that we are dealing with graph-like entities.)
- We then have the chance to explore the variety of ways in which we can talk about, and reason about, such structures — and hopefully we can find ways of doing so that are well behaved mathematically and accord with our intuitions about various problems. Classic example: "Logic of Time", by Johan van Benthem.

So what was Prior's contribution?

- Modern modal logic a valuable tool for the reasons discussed in the first lecture.
- Prior's insight was that formulas could be used as terms. We do not need to stick to the traditional logical categories when engaged in applied logic.
- Ultimately, this has showed us how to cut the cake of expressivity along very different lines. There are more interesting options for talking about relational structures than hitherto expected. We have a larger playground.

Arguably useful

- Naming using formulas corresponds naturally to design choices made in fields unaware that they were doing modal logic, such as feature logic and description logic.
- Allows a unification of seemingly incompatible views: Prior and Reichenbach not only can live together, arguably they should.
- Indeed, they can live happily together with Kaplan and Kamp too: all this fits together nicely with a theory of indexicals ("Tense, Temporal reference, and Tense Logic", Patrick Blackburn, Journal of Semantics, 1994).

Theoretical insight

- We have a hierarchy of expressivity options: we can carve out the bisimulation invariant fragment (modal logic), the bisimulation with constants invariant fragment (basic hybrid logic), the generated submodel invariant fragment (downarrow).
- And once we reach the bisimulation-with-constants (the basic hybrid language) invariant fragment, the metatheory starts to stabilize: we can import first-order techniques directly down into a decidable PSPACE logic and gain general results of a type that can't be obtained in the orthodox setting.
- Hybrid logic is important because it shows it is possible to to import first-order techniques wholesale into a modal setting. A genuine hybrid of two important perspectives is not only possible, it turns out to be natural.