Graph Games and Logic Design (PART I)

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Computational Logic and Applications Winter School
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Outline

Games and Logic

"Games are useful as intuitive guides in proofs and constructions"

Jouko Väänänen 2011, *Models and Games*.

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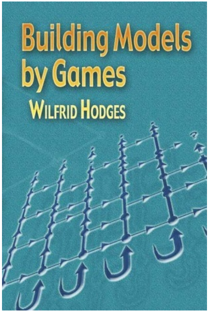
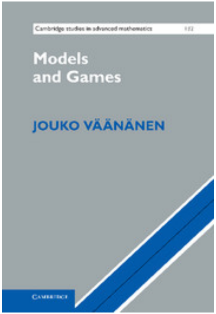
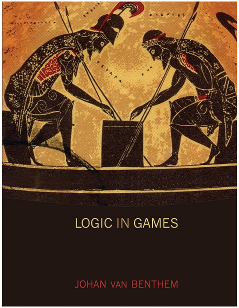
E.g.:

- Game semantics for logic
- Game used for determining whether two structures are equivalent.

Games and Logic







Logic in Games

Graph Games I: the Sabotage Game

Traveler plays against Demon. Demon moves first and cuts one edge per turn. Traveler starts at vertex 1 and moves along edges, trying to reach the goal at vertex 4. Which player has a winning strategy?

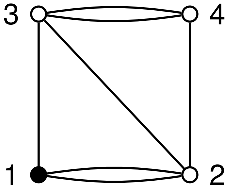


Figure: A simple sabotage game.

Sabotage Game: Relevant Features

- Undirected graph vs. directed graph.
- Multigraph (parallel edges allowed).
- The game models a standard computational (search) task under adverse circumstances. Without Demon, Traveler faces a graph reachability task of finding a path from the initial position to the goal region.
- Demon cut edges globally (i.e., delete any edge in the graph). ‘Local sabotage’ would cut edges only at the current position of Traveler.

Research on Sabotage Modal Logic

Home › Mechanizing Mathematical Reasoning › Chapter

An Essay on Sabotage and Obstruction

Chapter
pp 268–276 | Cite this chapter

Mechanizing Mathematical Reasoning

Johan van Benthem

Access this chapter

Home › FST TCS 2003: Foundations of Software Technology and Theoretical Computer Science › Conference paper

Model Checking and Satisfiability for Sabotage Modal Logic

Conference paper
pp 302–313 | Cite this conference paper

FST TCS 2003: Foundations of Software Technology and Theoretical Computer Science
(FSTTCS 2003)

Christof Löding & Philipp Rohde

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Home › Logic, Rationality, and Interaction › Conference paper

Sabotage Modal Logic: Some Model and Proof Theoretic Aspects

Conference paper | First Online: 19 November 2015
pp 1–13 | Cite this conference paper

Logic, Rationality, and Interaction
(LORI 2015)

Guillaume Aucher, Johan van Benthem & Davide Grossi

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Research on Sabotage Modal Logic

Volume 30, Issue 3
April 2020

JOURNAL ARTICLE

Losing connection: the modal logic of definable link deletion

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Dazhu Li

Journal of Logic and Computation, Volume 30, Issue 3, April 2020, Pages 715–743, https://doi.org/10.1093/logcom/exz036

Published: 15 April 2020 **Article history**

Volume 34, Issue 2
March 2024

JOURNAL ARTICLE

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Sujata Ghosh, Shreyas Gupta, Lei Li

Journal of Logic and Computation, Volume 34, Issue 2, March 2024, Pages 399–427, https://doi.org/10.1093/logcom/exad018

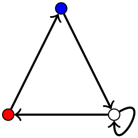
Published: 24 August 2023 **Article history**

When Random Graphs Are Safe for Travel: A Note on the Sabotage Game

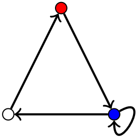
Krzysztof Mierzewski

Graph Game II: the Cop-and-Robber Game

Two players, Cop and Robber, move on the vertices of the graph. Cop aims to catch Robber; Robber aims to avoid capture. Does Cop have a strategy to capture Robber?



An Illustration



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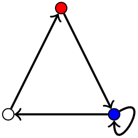
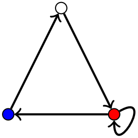


Does Cop have a strategy to capture Robber?

☞ Robber will not choose to enter the self loop.

An Illustration



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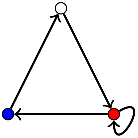
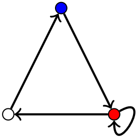


Does Cop have a strategy to capture Robber?

☞ Cop just waits in the self loop.

An Illustration



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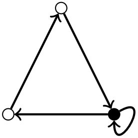

Does Cop have a strategy to capture Robber?

☞ Cop just waits in the self loop.

Studies in Math and CS

The Cop-and-Robber game has been extensively studied from algorithmic and combinatorial perspectives, e.g.

- Research questions: How many moves does it take for the cops to catch the robbers, given specific starting positions and graph structures? What is the minimum number of cops needed to capture the robbers on a specific graph?

- M. Aigner and M. Fromme (1984) A game of cops and robbers. *Discrete Applied Mathematics*, 8: 1-12
- A. Bonato and R. J. Nowakowski (2011) *The game of cops and robbers on graphs*. Student Mathematical Library, Vol. 61. American Mathematical Society
- W. B. Kinnersley (2018) Bounds on the length of a game of cops and robbers. Discrete Mathematics, 341(9): 2508-2518

From a Modal Logical Perspective

The Cop-and-Robber game has attractive analogies with modal logic.

$$\begin{aligned} \text{Game Board} &\Leftrightarrow \text{Possible States} \\ \text{Players' Moves} &\Leftrightarrow \text{Two Modalities} \\ \text{Capture} &\Leftrightarrow \text{Relationship of Two States} \end{aligned}$$

Modal logic allows us to study players' reasoning, describe winning conditions, and understand the game better.

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Yaxin Tu
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Princeton University

Our Works

- A simple logic of the hide and seek game (with D. Li, S. Ghosh, and Y. Tu, 2021; 2023.)
- Logic of the hide and seek game: Characterization, axiomatization, decidability (by Q. Chen and D. Li, DaLí Proceedings 2023.)
- Hybrid logic of the hide and seek game, *Studia Logica*, 2024 (with D. Li and K. Sano)
- Modal substitution logic (with Y. Tu, D. Li, and S. Ghosh, under submission, 2025.)
- Reasoning under uncertainty in the game of Cops and Robbers, *Synthese*, 2025, (with D. Li and S. Ghosh)

Modal Logic of Cop-and-Robber

We proposed a two-dimensional modal logic $\mathbf{LCR}$ that can study the interaction between two players on the graph.

Definition 1 (Language)

Let P_R and P_C denote two countable sets of propositional variables. The language is given as follows:

$$\varphi ::= p_R \mid p_C \mid I \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle R\rangle\varphi \mid \langle C\rangle\varphi$$

where $p_R \in \mathsf{P}_R$, $p_C \in \mathsf{P}_C$, and I is a propositional constant.

$\top, \bot, \to, \vee$ are defined as usual.
Also, $R\varphi := \neg\langle R\rangle\neg\varphi$ and $C\varphi := \neg\langle C\rangle\neg\varphi$

LCR: Models

Models $M = (W, R, V)$ are defined as follows:

- W is a non-empty set,
- $R \subseteq W \times W$ is a binary relation,
- $V : \mathsf{P}_R \cup \mathsf{P}_C \to \mathcal{P}(W)$ is a valuation function.

$M, s, t \models p_R, p_R \in \mathsf{P}_R$	$\Leftrightarrow$	$s \in V(p_R)$
$M, s, t \models p_C, p_C \in \mathsf{P}_C$	$\Leftrightarrow$	$t \in V(p_C)$
$M, s, t \models I$	$\Leftrightarrow$	$s = t$
$M, s, t \models \neg\varphi$	$\Leftrightarrow$	$M, s, t \not\models \varphi$
$M, s, t \models \varphi \wedge \psi$	$\Leftrightarrow$	$M, s, t \models \varphi$ and $M, s, t \models \psi$
$M, s, t \models \langle R \rangle \varphi$	$\Leftrightarrow$	$\exists s' \in W$ s.t. Rss' and $M, s', t \models \varphi$
$M, s, t \models \langle C \rangle \varphi$	$\Leftrightarrow$	$\exists t' \in W$ s.t. Rtt' and $M, s, t' \models \varphi$

Identity cross dimensions!

Describing the Game

- Cop can win (finite cases) $\Leftrightarrow \bigvee_{n \leq |M|^2} (R\langle C \rangle)^n I$

  Although a game on a finite graph can run indefinitely, it must repeat a previous situation after $|M|^2$ rounds, implying the Cop cannot catch the Robber.

Describing the Game

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- The general setting calls for more powerful device involving substitutions: to determine the Cop's winning positions, we proceed as follows:

  $$(\text{Win}_C := R\langle C\rangle\ \text{Win}_C)^*$$

  this involves an iterative calculation for Win_C, initially defined as $\{(s, s) \mid s \textit{ is a state of the graph}\}$.

LCR-bisimulation

Definition 2

Let $M = (W, R, V)$ and $M' = (W', R', V')$ be models. We say that a non-empty relation $Z \subseteq (W \times W) \times (W' \times W')$ is *an* **LCR**-*bisimulation between* M *and* M', if the following conditions hold:

Atom: If $(s,t)Z(s',t')$, then for all $p \in P_C \cup P_R$, $M, s, t \models p$ iff $M', s', t' \models p$.

Meet: If $(s,t)Z(s',t')$, then $s = t$ if and only if $s' = t'$.

Zig$_R$: If $(s,t)Z(s',t')$ and $v \in R(s)$, then there is $v' \in R'(s')$ s.t. $(v,t)Z(v',t')$.

Zig$_C$: If $(s,t)Z(s',t')$ and $v \in R(t)$, then there is $v' \in R'(t')$ s.t. $(s,v)Z(s',v')$.

Zag$_R$: If $(s,t)Z(s',t')$ and $v' \in R'(s')$, then there is $v \in R(s)$ s.t. $(v,t)Z(v',t')$.

Zag$_C$: If $(s,t)Z(s',t')$ and $v' \in R'(t')$, then there is $v \in R(t)$ s.t. $(s,v)Z(s',v')$.

LCR: Technical Results

Theorem 3

$\mathbf{LCR}$ *can be translated into the 3 variable fragment of FOL.*

Theorem 4 (van Benthem Characterization)

A FOL-formula is equivalent to the translation of an $\mathbf{LCR}$*-formula* φ *iff it is invariant for* $\mathbf{LCR}$*-bisimulation.*

Theorem 5

$\mathbf{LCR}$ *does not have the tree model property. It lacks the finite model property, too.*

LCR: Technical Results

Theorem 6

The satisfiability problem for $\mathbf{LCR}$ *is undecidable, while the fragment without* I *is decidable.*

Although I looks simple, $I \to RC\langle R\rangle\langle C\rangle I$ can define grid structure, and we can encode the $\mathbb{N} \times \mathbb{N}$ tiling problem.

Theorem 7

Model checking for $\mathbf{LCR}$ *is* P*-complete.*

LCR: Technical Results

The proof system below is a sound and strongly complete calculus for the fragment without I:

Axiom schemes:	
(A1)	$\varphi \to (\psi \to \varphi)$
(A2)	$(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi))$
(A3)	$(\neg\psi \to \neg\varphi) \to (\varphi \to \psi)$
(K)	$X(\varphi \to \psi) \to (X\varphi \to X\psi)$, for $X \in \{C, R\}$.
($\mathtt{Red}_C$)	$C(\psi \vee \gamma) \leftrightarrow (C\psi \vee \gamma)$, where $\psi \in \mathcal{L}_C$ and $\gamma \in \mathcal{L}_R$.
($\mathtt{Red}_R$)	$R(\psi \vee \gamma) \leftrightarrow (\psi \vee R\gamma)$, where $\psi \in \mathcal{L}_C$ and $\gamma \in \mathcal{L}_R$.
Inference rules:	
(MP)	From φ and $\varphi \to \psi$, infer ψ.
(Nec)	From φ, infer $X\varphi$, for $X \in \{R, C\}$.

Key idea of the completeness proof: Split the language into two “isolated parts” for Cop and Robber respectively.

LCR: Technical Results

For the logic with I, we have a calculus in the setting with nominals and @-operators (cf. Sano, Liu and Li, 2024). The key part:

Interaction axioms for the hybrid product logic	
(Com@)	$@_i @_a \varphi \leftrightarrow @_a @_i \varphi$
(Com$\langle C\rangle @_a$)	$\langle C\rangle @_a \varphi \leftrightarrow @_a \langle C\rangle \varphi$
(Com$\langle R\rangle @_i$)	$\langle R\rangle @_i \varphi \leftrightarrow @_i \langle R\rangle \varphi$
(Red$@_i$)	$@_i a \leftrightarrow a$
(Red$@_a$)	$@_a i \leftrightarrow i$
Axioms for propositional variables	
(Red$@_i$Pr)	$@_i p_C \leftrightarrow p_C$
(Red$@_a$Pr)	$@_a p_R \leftrightarrow p_R$
Axioms and rules for I	
(RepI_1)	$@_i @_a I \wedge @_j @_a I \to @_i j$
(RepI_2)	$@_i @_a I \wedge @_i @_b I \to @_a b$
(Eq$I\langle\cdot\rangle$)	$@_i @_a I \wedge @_j @_b I \to (@_i \langle H\rangle j \leftrightarrow @_a \langle S\rangle b)$
(NameI_R)	From $@_j @_a I \to \varphi$, infer φ, where $j \in \mathsf{N}_R$ is fresh in φ.
(NameI_C)	From $@_i @_b I \to \varphi$, infer φ, where $b \in \mathsf{N}_C$ is fresh in φ.

More on decidability: the hybrid extension without I is still decidable.

@-operators are also a proposal for equality, e.g., $@_i j$ means $i = j$.

I VS @

$\Downarrow$ cross dimension $\quad$ $\Downarrow$ within a dimension

Implication of the Logical Results: An Example

Consider the result: The model-checking problem for $\mathbf{LCR}$ is P-complete. This implies,

Implication of the Logical Results: An Example

Consider the result: The model-checking problem for **LCR** is P-complete. This implies,

Given a model (graph) and a formula expressing a player's winning condition, an algorithm can determine in polynomial time whether the formula is satisfied. If so, the player can, in principle, win the game.

The complexity result of the model checking problem sets an upper bound.

Further Open Problems

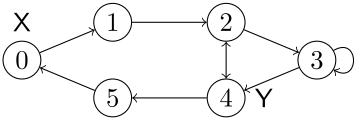
- (Finite) axiomatization of $\mathbf{LCR}$.
- The exact complexity of the satisfiability problem of $\mathbf{LCR}$.
- Baltag's questions: What would the logic look like if we stipulate that "knowing is winning" as the winning condition? Would the undecidability result be affected or improved? These questions have motivated the research project of modelling uncertainty of the game.

Uncertainty in the Cop-and-Robber game

- Graph games have been mainly studied as games of perfect information.
- In the Cop-and Robber game, players cannot observe the moves of their opponents and may be constrained by their sight or physical obstacles.
- Question: How can we model players' reasoning under uncertainty in graph games?

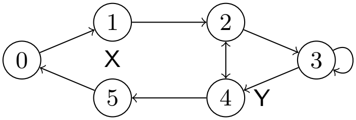
A Scenario with Restricted Observation

Initially, Cop X (female) is at 0, and Robber Y (male) is at 4. They know the graph structure and their own positions. A player can see the other if they are at the same position or at a vertex reachable by an arrow in either direction.



- Thus, X and Y do not know each other's exact positions.
- X knows that Y must be at 2, 3, or 4, while Y knows that X must be at 0 or 1.

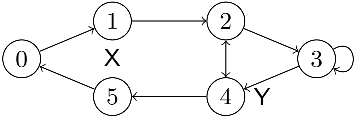
A Scenario with Restricted Observation



Players move along an arrow. Let X moves first to 1, we have:

X knows that Y is at 3 or 4 and Y knows that X is at 1.

A Scenario with Restricted Observation



Players move along an arrow. Let X moves first to 1, we have:

X knows that Y is at 3 or 4 and Y knows that X is at 1.

Next, let Y move to 5.

- X considers that after the movement, Y can only be at a successor of the previous possibilities, i.e., 2, 3, 4, or 5. However, since X can observe that Y is not at 2, she knows that that Y must be at 3, 4, or 5.
- Y still remembers that X is at 1.

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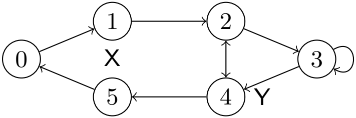

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- Y still remembers that X is at 1.

Finally, X moves to 2.

- X can see that Y is not at 3 or 4, and concludes that Y must be at 5 and wins.
- Y still knows the new position of X, i.e. the unique successor of 1, 2.

Game Design: Assumptions

Common knowledge: The graph structure, whose turn to move. In each round, Cop X moves first, and then Robber Y moves.

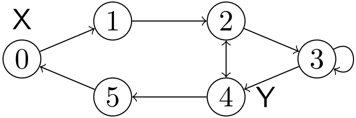
k-sight ability: Players always know their own positions, and if their positions are reachable within k steps via the arrows or their converse directions, then they know the positions of each other.

Winning condition: Fix a natural number $n \in \mathbb{N}$. Cop X wins iff the position of Robber Y is known by X within n rounds.

Logic Design: Basic Ideas

- We use values for nodes in a graph, which together with the binary relation give us a FOL structure.
- We use variables for players, then the current position of a player gives us the value of that variable.
- So, positions of all players can give us an assignment σ that assigns values to variables, say, if player x is at a, then $\sigma(x) = a$.

Revisiting the example, we have $\sigma(x) = 0$, $\sigma(y) = 4$.



Baltag (2016) To know is to know the value of a variable. *AiML 2016*.

Epistemic Logic for Cop-and-Robber (ELCR)

We fix a vocabulary $Voc = (Pred, Cons, Var)$:

- $Pred$ is a set of predicate symbols, containing a special binary relation R describing the edges of the graph of a game.
- $Cons$ is a non-empty, finite set of constants.
- $Var = \{x, y\}$ are the two variables, meaning the players.

Definition 8

Formulas in *the language* $\mathcal{L}$ *for* ELCR are defined as follows:

$$\mathcal{L}_{\mathsf{B}} \ni \alpha ::= P\boldsymbol{t} \mid t_1 \equiv t_2 \mid \neg\alpha \mid (\alpha \wedge \alpha)$$
$$\mathcal{L}_{\mathsf{BD}} \ni \psi ::= \alpha \mid \neg\psi \mid (\psi \wedge \psi) \mid [z]\psi$$
$$\mathcal{L} \ni \varphi ::= \psi \mid K_z t \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_z\psi \mid [z]\varphi$$

where $t, t_1, t_2 \in Cons \cup Var$ are terms, $\boldsymbol{t}$ is a tuple of terms, $P \in Pred$ is a predicate symbol, and $z \in Var = \{x, y\}$ is a variable.

$\langle K_z\rangle\varphi := \neg K_z\neg\varphi$, $\langle z\rangle\varphi := \neg[z]\neg\varphi$, $K_z\mathcal{T} := \bigwedge_{t\in\mathcal{T}} K_z t$ $(\mathcal{T} \subseteq Cons \cup Var)$.

Models

Definition 9

A k-*sight model for* ELCR is a tuple $M = (\mathbf{D}, \mathbf{I}, \Sigma, \sim)$, where

- $\mathbf{D}$ is a non-empty, finite set of values (also called vertices or positions).
- $\mathbf{I}$ is the interpretation function s.t.
  - $\mathbf{I}(P) \subseteq \mathbf{D}^m$ is an m-ary relation on $\mathbf{D}$. $\mathbf{I}(R)$ is a binary relation on $\mathbf{D}$ s.t. for any $s \in \mathbf{D}$, there is some $t \in \mathbf{D}$ s.t. $(s, t) \in \mathbf{I}(R)$.
  - $\mathbf{I}(c) \in \mathbf{D}$, for $c \in Cons$. Moreover, for each $s \in \mathbf{D}$, there is $c \in Cons$ s.t. $\mathbf{I}(c) = s$.
- $\Sigma \subseteq \mathbf{D}^{Var}$ is a non-empty set of *situations* of the players' positions, also called *assignments*.
- $\sim_z \subseteq \Sigma \times \Sigma$ is an equivalence relation s.t. for all $\sigma, \sigma' \in \Sigma$, if $\sigma \sim_z \sigma'$, then for all $z' \in Var$ with $\sigma(z') \in \mathbb{D}^k(\sigma(z))$, $\sigma(z') = \sigma'(z')$.

Intuitively, $u \in \mathbb{D}^k(v)$ means that from v one can reach vertex u within k steps via $\mathbf{I}(R)$ or its converse relation, which can be defined precisely.

Truth Conditions for the Static Part

Given a model $M = (\mathbf{D}, \mathbf{I}, \Sigma, \sim)$ and a situation $\sigma \in \Sigma$, we use $t^{(\mathbf{I},\sigma)}$ for the value of $t \in Var \cup Cons$.

Truth conditions (without $[z]\varphi$):

$$M, \sigma \vDash P(t_1, \ldots, t_n) \quad \text{iff} \quad (t_1^{(\mathbf{I},\sigma)}, \ldots, t_n^{(\mathbf{I},\sigma)}) \in \mathbf{I}(P)$$

$$M, \sigma \vDash t_1 \equiv t_2 \quad \text{iff} \quad t_1^{(\mathbf{I},\sigma)} = t_2^{(\mathbf{I},\sigma)}$$

$$M, \sigma \vDash K_z t \quad \text{iff} \quad \text{for all } \sigma' \sim_z \sigma,\ t^{(\mathbf{I},\sigma)} = t^{(\mathbf{I},\sigma')}$$

$$M, \sigma \vDash K_z \varphi \quad \text{iff} \quad \text{for all } \sigma' \sim_z \sigma,\ M, \sigma' \vDash \varphi$$

Some Validities

(1) $t_1 \equiv t_2 \to (\alpha \leftrightarrow \alpha[t_1/t_2])$, given $\alpha \in \mathcal{L}_\mathsf{B}$ — Closed for B-formulas

(2) $\bigvee_{c \in Cons} z \equiv c$, $z \in \{x, y\}$ — At some where

(3) $t \equiv c \to (K_z t \leftrightarrow K_z t \equiv c)$, $z \in \{x, y\}$, t is a term. — De re knowledge

(4) $(K_z t_1 \wedge K_z t_2 \wedge R t_1 t_2) \to K_z R t_1 t_2$ — Knowing the graph

(5) $\mathsf{D}^k zt \to K_z t$ — k-sight ability

(6) $K_x y = \mathcal{T} \to (K_x \alpha \leftrightarrow \bigwedge_{c \in \mathcal{T}} \alpha[c/y])$, where $\mathcal{T} \subseteq Cons$ and $\alpha \in \mathcal{L}_\mathsf{B}$. — Grounded knowledge

$\mathsf{D}^k zt$ is a predicate symbol for $\mathbb{D}^k$, which is definable with R, due to the finiteness of $Var \cup Cons$:

$$\mathsf{D}^0 t_1 t_2 := t_1 \equiv t_2$$

$$\mathsf{D}^{n+1} t_1 t_2 := \mathsf{D}^n t_1 t_2 \vee \bigvee_{t \in Cons \cup Var} (\mathsf{D}^n t_1 t \wedge (R t t_2 \vee R t_2 t))$$

$K_x y = \mathcal{T}$: $\mathcal{T}$ is the collection of y's possible positions considered by x. It is also definable in the language since $Cons$ is finite.

Dynamic Updates

Consider $[z]\varphi$, we need to capture the following:

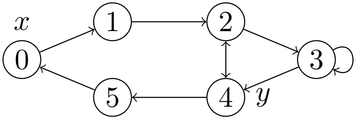
(1) Player z moves, so the situation of the game changes.

(2) The movements affect all players' knowledge.

One assumption: when a player acts, the other would know.

For how this gets done, see Dazhu Li's presentation this afternoon.

Example Revisited



We use the ordered pairs $[\sigma(x), \sigma(y)]$ for situations σ. e.g., $[0, 4]$ means $\sigma(x) = 0$ and $\sigma(y) = 4$.

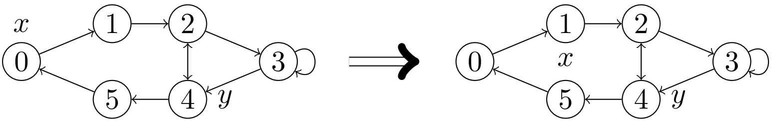
The original class of situations is

$$\Sigma_0 = \{[0, 4], [0, 3], [0, 2], [1, 4]\},$$

with the actual one in red.

A player $z \in \{x, y\}$ cannot distinguish two situations iff, the positions of z in the two situations are the same.

Example Revisited: Stage 1



The two players are not in each other's sight, and according to our definition, we have the following:

$$\Sigma_1 = \{\sigma \in \mathsf{R}^x(\Sigma_0) \mid \sigma(x) \notin \mathbb{D}^1(\sigma(y))\} = \{[1,4],\ [1,3]\}$$

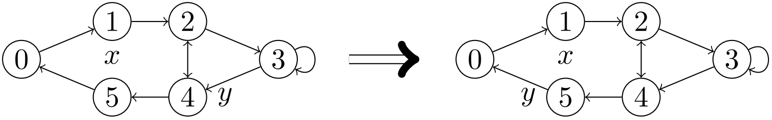
$$(\Sigma_0 = \{[0,4],[0,3],[0,2],[1,4]\})$$

Formally,

$$\neg K_x y \wedge K_x(y \equiv 3 \vee y \equiv 4) \quad \text{and} \quad K_y x.$$

Example Revisited: Stages 2 and 3

Robber y moves to 5:



Again, they are not in the sight of each other.

$$\Sigma_2 = \{\sigma \in \mathsf{R}^y(\Sigma_1) \mid \sigma(x) \notin \mathbb{D}^1(\sigma(y))\} = \{[1,5],\ [1,3],\ [1,4]\}.$$

$$(\Sigma_1 = \{[1,4],[1,3]\})$$

Now, y still knows where x is, but x has more uncertainties:

$$K_x(y \equiv 3 \vee y \equiv 4 \vee y \equiv 5) \quad \text{and} \quad K_y x \equiv 1.$$

Finally, x moves to 2:

$$\Sigma_3 = \{\sigma \in \mathsf{R}^x(\Sigma_2) \mid \sigma(x) \notin \mathbb{D}^1(\sigma(y))\} = \{[2,5]\}.$$

They both know the actual case.

Technical results

Theorem 10

The calculus is sound and strongly complete with respect to the static part of ELCR*.*

Theorem 11

The full language can be axiomatized using reduction axioms for dynamic operators, and it is both sound and strongly complete.

Theorem 12

The satisfiability problem for the logic is decidable.

In response to Baltag's question: yes, the decidability result is indeed improved.

Conclusion

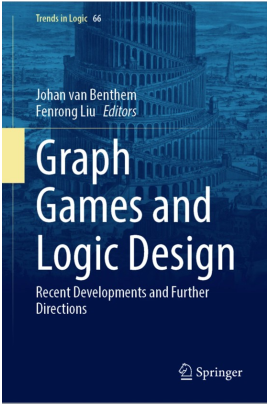
- There is a long tradition of interplay between logic and games. I presented developments in graph games and modal logic to highlight this fruitful interplay.
- The Cop-and-Robber game gives rise to a two-dimensional modal logic LCR, with strong expressive power but undecidability.
- When we shift to an epistemic perspective ”winning is knowing”, we obtain ELCR, which is decidable and still expressive enough to model complex reasoning with uncertainties.
- By changing the winning condition, we change the logical landscape. The logical results enhance our understanding of the games.

Future Directions

- More players with simultaneous movements
- Various kinds of abilities and knowledge of players, e.g., communication and distributed knowledge.
- Introducing other logical notions that are relevant to Cop-and-Robber, e.g., higher-order knowledge
- Connecting with graph theory more closely

This offers opportunities for collaborating with the Argentine School of Logicians!

Thanks!



I'll now hand it over to Johan van Benthem!

Welcome to Tsinghua Logic Center!

https://tsinghualogic.net/JRC/